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On Meeting the Provisions of the Clean Air Act

A general structure is proposed for determining a set of long-term (multi-year) source control measures which achieve specified levels of air quality for an airshed at least cost. Such a structure is useful in evaluating alternative air pollution abatement strategies from the standpoint of air quality and total cost. The theory is applied to the problem of evaluating sets of control actions over the period 1973-1975 in Los Angeles. Costs of achieving various levels of air quality in Los Angeles over this period are determined.

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SCOPE

The 1970 amendments to the Clean Air Act mandated clean air goals to be achieved in major urban areas. On June 15, 1973, the Administrator of the Environmental Protection Agency announced transportation control plans for 23 metropolitan areas which, if implemented, would result in substantial limitations in gasoline sales and inner city traffic in these areas by 1977. While the measures proposed would lead to reduced emissions of primary air pollutants, it is not clear that the particular elements of the control plan will lead to the desired levels of air quality and also do so at costs which are politically tolerable. A formalism which permits the systematic evaluation of the multitude of possible air pollution control strategies with respect to both air quality and total cost is highly desirable in developing implementation plans for urban regions. The presentation of such a formalism is the subject of this paper. The problem considered is to determine the set of control measures that minimizes the total cost of control while maintaining specified levels of air quality in each of a given number of years. It is assumed that an airshed model which is capable of relating atmospheric pollutant levels to source emission strengths is available. Control methods are considered for both fixed and mobile sources. The objective is to determine the least-cost allocation of existing control methods to sources in a given region such that prescribed air quality standards are not violated.

CONCLUSIONS AND SIGNIFICANCE

A general framework for the evaluation of air pollutant emission control strategies for an airshed is presented. The framework is developed so that it is applicable to an arbitrary number and type of sources, an arbitrary type of emission level/air quality model, and an arbitrary measure of air quality. Computational methods for determining the optimal set of control measures are briefly discussed. The theory is applied to the evaluation of air pollution control strategies for Los Angeles County for the period 1973 to 1975. The important primary pollutants in Los Angeles are nitric oxide and hydrocarbons, while air quality standards are based on atmospheric levels of nitrogen dioxide and ozone, both secondary pollutants. The least-cost control strategies are determined on the basis of air quality standards expressed in terms of the number of days per year that NO_2 and O_3 concentrations exceed current primary standards (0.25 and 0.10 ppm for one-hour average) in downtown Los Angeles. Use of these constraints enables use of an empirical emission level/air quality correlation developed by Trijonis (1972). It is shown that the number of days per year that the NO_2 and O_3 primary standards are violated can be reduced from the estimated 1975 level of 26 and 77 days, respectively, to 10 and 50 days at a cost of approximately 70 million dollars over the period 1973-1975.

An airshed system can be visualized to consist of a number of components: (1) Various pollutant-emitting sources, such as motor vehicles, power plants, industries, aircraft, etc. (2) Various chemical species, that is, pollutants. The primary pollutants, which are emitted directly from sources, consist predominantly of carbon monoxide (CO), hydrocarbons, nitrogen oxides (NO_x), SO_2, and particulate matter. The secondary pollutants, formed by atmospheric chemical reactions, consist of O_3, NO_2, sulfates, nitrates, and organic compounds. (3) A multitude of control methods for abating the emissions of the various sources. (For example, motor vehicle emissions can be abated through the use of evaporative control systems, catalytic mufflers, etc. Emissions from power plants can be controlled by burner modifications, substitution of natural gas for fuel oil, etc.) (4) Meteorological parameters consisting of wind speed and direction, stability conditions, radiation intensity, and the frequency of occurrence of various levels of each throughout the year.

A set of control measures applied to the sources will produce a given level of pollutant emissions having a certain spatial and temporal distribution. Associated with these emission levels will be a modified level of air quality. Conversely, for a desired level of air quality, there may exist many possible combinations of control measures that can produce the specified air quality. Therefore, the question arises as to which set of measures is, in some sense, the best.

We can envision the analysis of this question as being composed of two steps: establishment and evaluation of (1) the relationship between control methods and emission levels, and (2) the relationship between emission levels and air quality. The control method/emission level problem consists of determining the least-cost allocation of controls to achieve certain specified mass emission levels from a variety of sources. This problem has been considered extensively in the literature, and a number of formulations leading to mathematical programming solutions have been proposed (Kohn, 1969, 1970, 1971; Burton and Sanjour, 1970; Trijonis, 1972; Seinfeld and Kyan, 1971; Kyan and Seinfeld, 1972; Muller, 1973).

The emission level/air quality problem consists of the conversion of emission levels into spatial (and, perhaps temporal) distributions of airborne pollutant concentrations. Most of the reported studies have been concerned with air quality goals based on long-term averages (say, yearly). Several studies (for example, Farmer et al., 1970; Burton and Sanjour, 1970; Hamburg and Cross, 1971) have utilized the long-term Gaussian plume model (Martin, 1971) to relate point, line, and area source emission rates to annual average ground level pollutant concentrations. This model permits consideration of a single inert or multiple, noninteracting pollutants. When the long-term Gaussian plume model is used, the linear structure of the entire problem is preserved since the relationship between emission levels and air quality enters only as a linear constraint. Trijonis (1972) treated reactive photochemical pollutants by means of an empirical model based on correlations of past air monitoring data with estimated emission levels. Because the empirical model resulted in graphical relationships between emission levels and air quality, the overall problem was still amenable to solution by linear programming.

Most studies published to date have not included three major aspects:

1. The ability to determine strategies over a multi-year period,

2. A detailed consideration of control methods for mobile sources, and

3. The ability to include an arbitrary emission level/air quality model, either a long-term or short-term model, for both inert and reactive pollutants. More realistic air quality exercises will require consideration of each of the above aspects. The paper of Kyan and Seinfeld (1972) presents a formulation of the least-cost control problem which includes provisions for each of these aspects. However, the example considered in that paper was largely hypothetical since it related only to carbon monoxide control in the Los Angeles airshed.

Our objective here is two-fold. First, we present a formulation of the least-cost problem, a formulation representing a unification and generalization of the available treatments. Second, we apply the theory to the problem of determining least-cost controls for hydrocarbons and NO_x in Los Angeles County for the period 1973-5.

A GENERAL APPROACH FOR LEAST-COST AIR POLLUTION CONTROL STRATEGIES

In this section we propose a general framework for the evaluation of air pollutant emission control strategies for an airshed. The framework is developed so that it is applicable to an arbitrary number and type of sources, an arbitrary type of mathematical emission level/air quality model, and an arbitrary measure of air quality. Computational methods for determining the optimal set of control measures have been developed and are discussed in the next section.

Let us assume that we have an airshed for which the various polluting sources and their associated distribution of mass emission levels of air pollutants are known. In addition, we have categorized all available emission control methods and their costs. Finally, specified air quality criteria exist, together with a model which in some manner relates emission levels to air quality. The problem is to determine that combination of control measures, employed over a specified period of years, that will lead to prescribed levels of air quality at least cost.

For the mathematical formulation of the problem, the following definitions are used:

$s(t)$ = p-source vector in year t, with $s_j(t)$ being the units of source j in year t (for example, $s_1(1)$ may be the number of pre-1966 vehicles in the region in year 1)

p = total number of sources in the airshed

$E(t)$ = $m \times p$ matrix, with E_{ij} being the mass emission of pollutant per given time period from a unit of source j (for example, $E_{11}(1)$ may be the grams of CO emitted per day by a pre-1966 motor vehicle in year 1). $E^0(t)$ = emission level matrix in the absence of control

m = number of pollutants

$e(\xi, \tau; t)$ = m-emission rate vector, with $e_i(\xi, \tau; t)$ being the mass emission of species i per unit time from all sources at location ξ and time τ (during the day) in year t

[$E(t)$ accounts for the emissions from each type of source over the entire airshed, while $e(\xi, \tau; t)$ represents the emission rate of pollutants from all sources as a function of location and time. $E(t)$ will be required for control method evaluation while $e(\xi, \tau; t)$ is needed for the airshed simulation model.]

$d_{ij}(t)$ = number of units of control method j per unit of source i in year t (for example, $d_{11}(1)$ may be the number of emission control devices installed on pre-1966 motor vehicles in year 1)

q_j = number of control methods available for source j, $j = 1,2,\ldots,p$

$K = \sum_{i=1}^{p} q_i$ = total number of control methods available for all sources in the airshed

[There are two classes of control methods, those which can be instituted on a year-to-year basis independently of former years (for example, substitution of one fuel for another in power plants) and those which represent the installation of a piece of equipment expected to remain on the source for the life of the control device. We can partition the K $d_{ij}(t)$ values into those of the first and second categories, respectively. In order to represent compactly the $d_{ij}(t)$, we introduce the K-vector $w(t)$,

$$w(t) = \begin{bmatrix} \overline{w}(t) \\ \overline{\overline{w}}(t) \end{bmatrix}$$

where $\overline{w}$ is a $\overline{K}$-vector comprised of elements of the form $s_i d_{ij}$ for methods which can be instituted on a year-to-year basis, and $\overline{\overline{w}}$ is a $\overline{\overline{K}}$-vector comprised of elements of the form $s_i d_{ij}$ for methods which are of a semi-permanent nature. Note that $K = \overline{K} + \overline{\overline{K}}$.]

$R(t)$ = $m \times K$ matrix, with R_{ij} being the reduction in the mass emission of pollutant i per unit of control w_j in year t (for example, R_{11} may be the reduction in exhaust NO_x emissions for a 1966 vehicle upon installation of one capacitor-discharge-ignition optimization system)

$A(t)$ = $p \times K$ matrix with A_{ij} being the units of source i controlled by one unit of $w_j(t)$ in year t

[Corresponding to the partitioning of w into $\overline{w}$ and $\overline{\overline{w}}$, we can partition $R(t)$ and $A(t)$ into $\overline{R}$ and $\overline{\overline{R}}$ and $\overline{A}$ and $\overline{\overline{A}}$, as follows:

$$R(t) = \left[\overline{R} \mid \overline{\overline{R}}\right] \qquad A(t) = \left[\overline{A} \mid \overline{\overline{A}}\right]$$

$c_{ij}(t)$ = cost of one unit of control method j per unit of source i in year t

[As in the definition of $w(t)$, we can define $c(t)$ as

$$c(t) = \begin{bmatrix} \overline{c}(t) \\ \overline{\overline{c}}(t) \end{bmatrix}$$

where the ordering of $c(t)$ is the same as that of $w(t)$.]

$l(t)$ = M-vector of limited supply inputs in year t (for example, $l_1(1)$ may be the total amount of natural gas available in year 1 for the airshed)

$D(t)$ = $M \times K$ matrix, with D_{ij} being the amount of the ith limited supply input consumed by one unit of control method w_j in year t

$x(t)$ = m-vector with $x_i(t)$ being the total mass emission of pollutant i from all sources in the airshed per unit time in year t after institution of controls

$y^0(t)$ = m-vector, with $y_i^0(t)$ being the total mass emission of pollutant i from all sources in the airshed per unit time in year t with no controls in any of the years $1,2,\ldots,t$

$x^0(t)$ = m-vector, the uncontrolled total mass emission level per unit time for year t given controls existing in all prior years

$z(\xi, \tau; t)$ = m-vector airborne pollutant concentrations. The units of z are determined by the type of airshed model available, for example, for a dynamic airshed model z could be in units of ppm as a function of location and time (ξ, τ) during a typical day in year t, while for a statistical model, z could represent the number of days per year that a certain standard is violated in the airshed at location ξ and time τ

$g(z; t)$ = m'-vector, defining air quality, a prescribed function of the z_i for year t (for example, g_1 may be the hourly-average concentration of carbon monoxide at a certain location in the airshed). $g^*(t)$ is the maximum allowable value of $g(t)$

$\alpha(\xi)$ = p-vector of source distributions, where $\alpha_i(\xi)$ is the fraction of source i at location ξ in the airshed

$\beta(\tau)$ = p-vector of temporal source activities, where $\beta_i(\tau)$ is the fraction of daily activity of source i at time τ

Given these definitions, the activity of source i at location ξ and time τ during year t, $a_i(\xi, \tau; t)$, is given by $a_i(\xi, \tau; t) = \alpha_i(\xi)\ \beta_i(\tau)\ s_i(t)$.

Any mathematical air pollution model can be represented as a general functional relationship between emission levels and air quality,

$$F\,[z(\xi, \tau; t),\quad e(\xi, \tau; t)] = 0$$

that is, either an explicit or implicit relationship exists between e and z.

With the above definitions we are now in a position to formulate the least-cost air pollution control problem. In particular, we wish to minimize the total cost of control, that is,

Minimize

$$J = \sum_{t=1}^{T} \{\overline{c}^T(t)\overline{w}(t) + \overline{\overline{c}}^T(t)\overline{\overline{w}}(t)\} \qquad (1)$$

with respect to $w(t)$, $t = 1,2,\ldots, T$ subject to the state transition equations,

$$x(t) = x(t-1) + \overline{R}(t-1)\ \overline{w}(t-1) - R(t)\ w(t)$$
$$= x^0(t) - R(t)\ w(t) \qquad t = 1, 2, \ldots, T$$
$$x^0(1) = y^0(1)$$

which can be written as

$$x(t) + R(t)\ w(t) + \sum_{i=1}^{t-1} \overline{\overline{R}}(i)\ \overline{w}(i) = y^0(t) \qquad (2)$$

and the control constraints,

$$A(t)\ w(t) + \sum_{i=1}^{t-1} \overline{\overline{A}}(i)\ \overline{w}(i) \leq s(t) \qquad (3)$$

$$D(t)\ w(t) + \sum_{i=1}^{t-1} \overline{\overline{D}}(i)\ \overline{\overline{w}}(i) \leq l(t) \qquad (4)$$

$$w(t) \geq 0, \quad x(t) \geq 0 \qquad (5)$$

the air quality constraints,

$$g(z; t) \leq g^*(t) \qquad (6)$$

and constraints imposed by the emission level/air quality model,

$$F\,[z(\xi, \tau; t),\quad e(\xi, \tau; t)] = 0 \qquad (7)$$

The control measures are related to the emissions through

$$E(t)\ a(\xi, \tau; t) = e(\xi, \tau; t) \qquad (8)$$

$E(t)$ is computed by the relation

$$E_{ij}(t) = E^0_{ij}(t) - \sum_{k=1}^{q_j} r_{ikj}\, d_{jk}(t) \qquad (9)$$

$$i = 1, 2, \ldots, m$$

$$j = 1, 2, \ldots, p$$

where r_{ikj} is defined as the reduction in the emission of pollutant i per unit of control method k for source j.

The overall mathematical formulation of the problem is given by (1) to (9). In (1), the multi-year cost J is expressed as a sum of the cost of temporary controls $\bar{c}^T\bar{w}$ and of semi-permanent measures $\bar{\bar{c}}^T\bar{\bar{w}}$. Equation (2) states that the controlled emission level in year t, $x(t)$, plus the reduction in emission levels by all control methods instituted up to year t, must be equal to the uncontrolled emission level $y^0(t)$. Equation (3) specifies that all the sources controlled in any year should not exceed the total number of existing sources, while (4) stipulates that the maximum available quantities of limited supply inputs must not be exceeded in any year. The constraint that the air quality in each year must meet a prescribed standard is expressed by (6). Finally, the emission level/air quality model enters through (7) to (9), such that the choice of any set of control measures must give an emission level (and distribution) which ensures that the air quality, as predicted by the mathematical model (7), must comply with the air quality standard (6). In summary, the problem (1) to (9) states that the multi-year control cost J is to be minimized by choosing control methods $w_j(t)$, $j = 1,2,\ldots,K$, $t = 1,2,\ldots,T$, from all those possible such that the air quality standard is satisfied. We stress that it is not possible to consider each year as being independent of the others, since, as we have noted, many control methods involve the installation of equipment. The decision to install such equipment in an early year without considering the impact of that decision in later years may not yield a multi-year policy which is truly optimal. (Although this is a technicality which must be considered in properly formulating the multi-year control problem, in practical terms it will not have as much influence on the sequencing of control decisions as will the development of new control technology during the period over which the control is being applied.)

The problem, as formulated above, enables consideration of the spatial location of a source in determining the least-cost strategy for the entire region. Equations (7) to (9) express the relationship between the spatial and temporal distributions of both emissions and ambient pollutant concentrations. In order to allow for the possibility of controlling the same types of sources at different levels, depending on their locations, it is only necessary to index the sources separately in defining the original source vector $s(t)$. In doing so, the sources will be treated separately by the mathematical programming solution. At the least-cost solution the levels of control of the similar source type may indeed be different, depending on their respective locations.

IMPLEMENTATION OF THE GENERAL THEORY

The ultimate utility of the formulation presented in the last section depends on the development of computational means to determine the control vector $w(t)$, $t = 1,2,\ldots,T$. An important issue in this regard is that solution methods be relatively independent of the precise nature of the emission level/air quality model (7), although certain methods may be more or less advantageous depending on the form of (7).

In general, the formulations of interest to us:

1. Will involve linear objective functions (that is, cost relationships) although one may wish to consider the case of the unit cost of control varying with the level of control in a nonlinear fashion. In such a case, the objective function may consist of a set of linear fragments (piece-wise linear) or a quadratic relationship.

2. Will incorporate continuous variables. This is true because, where variables are integer, they generally involve numbers sufficiently large that they may be regarded as continuous for the purposes of calculation. However, there may be exceptions, such as control methods which involve a choice of installing or not installing a piece of equipment on a small number of source units. In such cases one may have to examine the use of mixed integer programming techniques.

3. Will often involve nonlinear emissions/air quality relations as constraints, particularly for short-term air quality restrictions. However, some of the simpler emissions/air quality relations will permit the use of linear programming.

As some of the main computational difficulties are those associated with the inclusion of the emission level/air quality relation (7) in the formulation, we wish to discuss these constraints in more detail. If (7) is linear or can be approximated by linear relationships in the vicinity of $g(t) = g^*(t)$, then (1) to (9) reduces to a single linear programming problem. In general, however, the emission level/air quality model is nonlinear.

There are basically two types of emission level/air quality models that can be developed for a region: (1) a physical dynamic simulation model, and (2) an empirical model based on the Gaussian plume equation or past air quality data. The former describes the behavior of pollutants over a period of several hours to several days and is based on the solution of equations which include relevant aspects of advection, diffusion, and chemical reaction (Seinfeld et al., 1972). The predictions of such a model are in the form of the spatial and temporal distributions of pollutants in the airshed. Meteorological variables, such as wind velocities and temperature structure, enter the model as inputs, specified from available measurements or predictions. An empirical model, on the other hand, is generally based on correlations of pollutant concentration levels with estimated emission levels. Meteorological parameters may enter either explicitly (as in Gaussian plume models used with a wind rose) or implicitly (as in direct correlations of pollutant levels with source strengths). Such models are normally based on a seasonal or yearly time period. The typical output of such a model is the probability that a certain air quality standard will be violated a given number of days per year if total daily emissions are at some specified level.

The advantage of a physical simulation model is that it is capable of predicting concentration levels for a variety of meteorological conditions and for any specified spatial and temporal distribution of emissions. The main disadvantage of such a model is that the solution of the governing equations often requires a large amount of computation. An empirical model has the desirable feature of being represented by algebraic equations or tabulated probabilities and therefore requires only a very small amount of computing in its use. The drawbacks of such a model are, however, substantial. Those models that do exist are generally based on correlation of air quality at one location with total daily emissions. Those that have spatial resolution are generally incapable of handling pollutants which react chemically.

In general, the type of ambient air quality standards that must be met will determine the nature of the constraints, (6) and (7). For example, an air quality standard based on a one-hour average will generally require a dynamic simulation model whereas one based on an annual average will not. If more than one type of air quality standard exists for the particular pollutant under

TABLE 1. INVENTORY OF SOURCES TO BE CONTROLLED IN LOS ANGELES COUNTY 1973-1975

Source no.	Description of source	Definition of source unit, SU	SU in 1973	SU in 1974	SU in 1975
1.	Nonpower plant large boilers, >30 MB.t.u./hr.	1 boiler	130	135	140
2.	Medium size boilers, 2 to 30 MB.t.u./hr.	1 boiler	5,410	5,630	6,000
3.	Large refinery heaters, ≥90 MB.t.u./hr.	1 heater	60	60	60
4.	Small refinery heaters, <90 MB.t.u./hr.	1 heater	160	160	160
5.	Rule-68-complying large power plant boilers,* 180-350 megawatt	1 boiler	8	8	8
6.	Nonrule-68-complying large power plant boilers,* 220-480 megawatt	1 boiler	8	8	8
7.	Small power plant boilers, 10-175 megawatt	1 boiler	37	37	37
8.	Large stationary internal combustion engines, ≥300 HP	1 engine	140	140	140
9.	Small compressor engines, <300 HP	1 engine	360	360	360
10.	Underground service station tanks	1 tank	32,000	33,000	34,000
11.	Service station, automobile tank filling	1 station	10,800	11,000	11,300
12.	Surface coating operations resulting in emission of reactive hydrocarbons	1 ton/day of emitted reactive	47	49	51
13.	Degreasers	1 ton/day of reactive organic solvent used for degreasing	24	25	26
14.	Dry cleaners using petroleum based solvents	1-day cleaning plant	25	25	25
15.	Pre-1966 motor vehicle exhaust emissions	1 vehicle	1.69×10^6	1.38×10^6	1.10×10^6
16.	Pre-1966 motor vehicle evaporative emissions	1 vehicle	1.69×10^6	1.38×10^6	1.10×10^6
17.	1966-1969 motor vehicle exhaust emissions	1 vehicle	1.49×10^6	1.45×10^6	1.38×10^6
18.	1966-1969 motor vehicle evaporative emissions	1 vehicle	1.49×10^6	1.45×10^6	1.38×10^6
19.	1970 model motor vehicle	1 vehicle	0.40×10^6	0.40×10^6	0.39×10^6
20.	1971-1974 model year fleet vehicles suitable for conversion to gaseous fuels	1 vehicle	1.76×10^5	2.58×10^5	3.45×10^5
21.	Jet aircraft—JT8D engines†	1 engine	2,500	2,500	2,500
22.	Jet aircraft—other engines	1 engine	2,900	2,900	2,900
23.	Piston aircraft engines registered in Los Angeles County	1 engine	6,350	6,650	7,000

* Los Angeles County Rule 68 provides for a two-step reduction in allowable emissions of oxides of nitrogen (NO_x) from fuel burning equipment producing more than 1,775 million B.t.u/hr. The rule specifies that the maximum ppm by volume of NO_x in the effluent gases from gas-fired equipment must be 225 ppm after December 31, 1971, and 125 ppm after December 31, 1974, and that the maximum ppm in oil-fired equipment must be 325 ppm after December 31, 1971, and 225 ppm after December 31, 1974.

† The JT8D engine is the standard jet engine on a medium range jet transport, such as the Boeing 727 and the Douglas DC-9.

consideration, then it is advantageous to conduct a preliminary analysis to determine whether each standard will influence the optimal solution or if one standard is the controlling factor.

AIR POLLUTION CONTROL FOR THE LOS ANGELES AIRSHED 1973-5

In this section we present a study of the control of nitrogen oxide and hydrocarbon emissions in Los Angeles County from 1973 to 1975. Aside from nitrogen oxides (denoted NO_x, although virtually all of the NO_x emitted from sources is in the form of nitric oxide, NO) and hydrocarbons, carbon monoxide is an important primary pollutant in Los Angeles. The undesirable characteristics of photochemical smog, such as ozone, eye irritation, and visibility limitation do not depend on CO levels. For this reason and also because control techniques for hydrocarbon emissions from motor vehicles will also result in simultaneous control of CO emissions, we consider here only control of hydrocarbon and NO_x emissions.

The current California air quality standards for oxidant (principally ozone) and NO_2 are that oxidant concentrations should not exceed 0.1 ppm for more than one hour and that NO_2 concentrations should not exceed 0.25 ppm for more than one hour. These air quality standards can be used as a basis for control strategy evaluation in either of two ways. First, we can specify that these standards are not to be violated more than a certain number of days per year. Second, we can specify that these standards are not to be violated anywhere in the urban area during a day with typical summertime meteorology. Clearly, these two alternative ways of considering the air quality constraint give rise to different needs for an emission level/air quality model. In the former case, we need a model giving the probability of occurrence of certain concentration levels over a seasonal or yearly period if emissions are at certain levels. In the latter case, a dynamic simulation model is called for [for example, see Reynolds et al. (1973abc) and Roth et al. (1973)]. In this study we concentrate on air quality constraints of the first type.

An empirical model based on correlation of the maximum daily one-hour average concentrations of NO_2 and O_3 at the downtown Los Angeles Air Pollution Control District monitoring station with total daily emissions of reactive hydrocarbons (RHC) and NO_x has been developed by Trijonis (1972). The net result of this model is shown in Figure 1, in which the number of days per year that both the NO_2 and O_3 standards (0.25 ppm and 0.10 ppm for one-hour averages, respectively) at the downtown station can be expected to be violated are plotted as a function of the daily emissions of RHC and NO_x in Los Angeles County. Figure 1 is based on an analysis of air quality measurements at the downtown station from 1965-1969. It should be noted that the hydrocarbon axis of Figure 1 differs somewhat from that originally determined by Trijonis because the reactive hydrocarbon emission estimates used by Trijonis were considerably lower than those now felt to be applicable.

In using Figure 1 we make the following definitions:

$g_1(x_1(t))$ = number of days per year the midday NO_2 concentration in downtown Los Angeles exceeds 0.25 ppm for one hour

$g_2(x_1(t), x_2(t))$ = number of days per year the O_3 concentration in downtown Los Angeles exceeds 0.10 ppm for one hour

$x_1(t)$ = emissions of NO_x from all sources in the basin in year t, tons/day

$x_2(t)$ = emissions of RHC from all sources in the basin in year t, tons/day

We let t = 1,2,3 correspond to 1973, 1974, 1975. Then the three prescribed values of $g^*(t)$, t = 1,2,3 constitute an air quality path defining the levels of air quality to be achieved in each of the three years. Each value of $g^*(t)$ specifies a point on Figure 1 which is the vertex of the region of admissible x_1 and x_2 values in year t, as shown

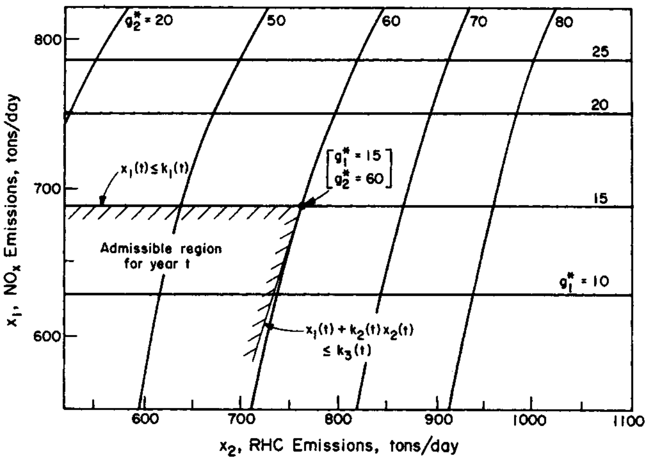

Fig. 1. Number of days per year that midday NO_2 and O_3 concentrations in downtown Los Angeles exceed 0.25 ppm and 0.10 ppm, respectively, for one hour as a function of the total daily emissions of reactive hydrocarbons (RHC) and oxides of nitrogen (NO_x) in Los Angeles County. Source: Trijonis (1972).

TABLE 2. ESTIMATED EMISSIONS OF REACTIVE HYDROCARBONS AND OXIDES OF NITROGEN FROM UNCONTROLLED SOURCES IN LOS ANGELES COUNTY, TONS/DAY*

Source	Reactive Hydrocarbons (RHC)				Oxides of Nitrogen (NO_x)			
no.	1969	1973	1974	1975	1969	1973	1974	1975
1	0	0	0	0	21.6	26	27	28
2	0	0	0	0	31.8	36.8	38.3	40.8
3	0	0	0	0	14.4	14.4	14.4	14.4
4	0	0	0	0	9.6	9.6	9.6	9.6
5	0	0	0	0	108.0	28.95	29.7	30.4
6	0	0	0	0	65.7	72.4	74.2	76
7	0	0	0	0	19.8	21.9	22.4	23.0
8	0	0	0	0	25.2	25.2	25.2	25.2
9	0	0	0	0	6.84	6.84	6.84	6.84
10	23.4	25.7	26.3	26.9	0	0	0	0
11	46.3	53	54	54.5	0	0	0	0
12	154	176	184	191	0	0	0	0
13	20.4	21.6	22.5	23	0	0	0	0
14	18.75	18.75	18.75	18.75	0	0	0	0
15	450.6	178.7	136.5	105.1	212.1	84.1	64.3	49.5
16	242.9	136.4	111.4	88.8	0	0	0	0
17	404.8	109.1	91.1	70.0	190.5	211.2	176.4	143.3
18	96	120.2	117.8	111.4	0	0	0	0
19	—	29.2	24.2	22.5	—	82.2	73.2	59.3
20	—	19.2	28.6	38.2	—	32.5	48.4	64.5
21	14.0	14.0	14.0	14.0	3.0	3.0	3.0	3.0
22	4.06	4.06	4.06	4.06	4.93	4.93	4.93	4.93
23	10.5	12.25	12.83	13.5	3.27	3.81	4.0	4.2
Total	1,485.7	918.2	846.1	781.8	716.7	663.8	621.8	582.9

* 1 ton = 907 kg.

in Figure 1. We can expect the optimally controlled emission levels to be as close as possible to the point defined by $g^*(t)$.

We note that the iso-air quality curves bounding an admissible region are approximately straight, especially in the vicinity of the vertex point where the two iso-air

Table 3. Control Methods, Costs and Emission Reduction Characteristics

Control no. (Element of w)	Description of one unit of control method	Control applicable to source	Cost per unit of control, \$/yr.	Source units per unit of control	Limited supply inputs per unit of control	RHC 1973	RHC 1974	RHC 1975	NO_x 1973	NO_x 1974	NO_x 1975
1	Low excess air firing (LEAF) to one large boiler	1	1,000	1	0	0	0	0	0.08	0.08	0.08
2	LEAF and flue as recirculation (FGR) to one large boiler	1	10,500	1	0	0	0	0	0.14	0.14	0.14
3	LEAF to one medium boiler	2	1,270	1	0	0	0	0	2.7×10^{-3}	2.7×10^{-3}	2.7×10^{-3}
4	LEAF and FGR to one medium boiler	2	2,700	1	0	0	0	0	4.7×10^{-3}	4.7×10^{-3}	4.7×10^{-3}
5	LEAF to one large heater	3	950	1	0	0	0	0	0.096	0.096	0.096
6	LEAF to one small heater	4	1,900	1	0	0	0	0	0.024	0.024	0.024
7	Substitution of 2.6×10^6 equivalent barrels/yr. of gas for fuel oil	5	0	1	2.6×10^6	0	0	0	1.21	1.24	1.27
8	Substitution of 3.9×10^6 equivalent barrels/yr. of gas for fuel oil	6	0	1	3.9×10^6	0	0	0	3.03	3.10	3.17
9	Advanced combustion modification	6	2.2×10^5	1	0	0	0	0	3.62	3.71	3.8
10	Advanced combustion modification and substitution of 3.9×10 equivalent barrels/year of gas for fuel oil	6	2.2×10^5	1	3.9×10^6	0	0	0	5.44	5.57	5.7
11	LEAF for one small power plant boiler	7	14,000	1	0	0	0	0	0.181	0.186	0.19
12	LEAF and FGR for small power plant boiler	7	51,000	1	0	0	0	0	0.296	0.303	0.31
13	Water injection (WI) or exhaust gas recirculation (EGR) to one large engine	8	680	1	0	0	0	0	0.14	0.14	0.14
14	WI or EGR to one small engine	9	190	1	0	0	0	0	0.014	0.014	0.014
15	Vapor recycle system (VRS) for one tank truck and 90 underground gas tanks	10	3,900	90	0	0.0709	0.0709	0.0709	0	0	0
16	VRS for one service station nozzle	11	330	1	0	3.732×10^{-3}	3.732×10^{-3}	3.732×10^{-3}	0	0	0
17	Further restrictions on reactive solvents in surface coating	12	4.5×10^6	191	0	132	138	143.3	0	0	0
18	Substitution of 0.75 tons/day of 1, 1, 1-trichloroethane for 1 ton/day of trichloroethylene	13	−3,280	1	0	0.9	0.9	0.9	0	0	0
19	Activated carbon system to one dry cleaner plant	14	3,300	1	0	0.712	0.712	0.712	0	0	0
20	Capacitor discharge-ignition optimization system (CDIOS) to one vehicle	15	9	1	0	6.34×10^{-5}	5.93×10^{-5}	5.7×10^{-5}	1.74×10^{-5}	1.63×10^{-5}	1.58×10^{-5}
21	EGR and control spark retardation (CSR) to one vehicle	15	45	1	0	1.58×10^{-5}	1.48×10^{-5}	1.43×10^{-5}	2.73×10^{-5}	2.56×10^{-5}	2.47×10^{-5}
22	Evaporative control retrofit (ECR) to one vehicle	16	78	1	0	6.86×10^{-5}	6.86×10^{-5}	6.86×10^{-5}	0	0	0
23	CDIOS to one 1966-1966 vehicle	17	1	1	0	7.32×10^{-6}	6.28×10^{-6}	5.36×10^{-6}	7.79×10^{-5}	6.69×10^{-5}	5.71×10^{-5}
24	Vacuum spark advance disconnect and tuning adjustment	17	6	1	0	2.19×10^{-5}	1.89×10^{-5}	1.61×10^{-5}	5.66×10^{-5}	4.86×10^{-5}	4.15×10^{-5}
25	ECR to one 1966-1969 vehicle	18	50	1	0	6.86×10^{-5}	6.86×10^{-5}	6.86×10^{-5}	0	0	0
26	CDIOS to one 1970 vehicle	19	0	1	0	0	0	0	11.28×10^{-5}	9.88×10^{-5}	8.36×10^{-5}
27	Conversion of one fleet vehicle to operate on natural gas	20	30	1	22.5	10.41×10^{-5}	10.41×10^{-5}	10.41×10^{-5}	14.8×10^{-5}	14.8×10^{-5}	14.8×10^{-5}
28	Conversion of one fleet vehicle to operate on liquid propane gas	20	130	1	0	9.08×10^{-5}	9.08×10^{-5}	9.08×10^{-5}	13.12×10^{-5}	13.12×10^{-5}	13.12×10^{-5}
29	Combustion chamber redesign on one JT8D engine	21	4,000	1	0	5.32×10^{-3}	5.32×10^{-3}	5.32×10^{-3}	-2.4×10^{-4}	-2.4×10^{-4}	-2.4×10^{-4}
30	Combustion chamber redesign on one non-JT8D engine	22	4,000	1	0	1.33×10^{-3}	1.33×10^{-3}	1.33×10^{-3}	-3.4×10^{-4}	-3.4×10^{-4}	-3.4×10^{-4}
31	After-burner on one piston aircraft engine	23	350	1	0	1.45×10^{-3}	1.45×10^{-3}	1.45×10^{-3}	0	0	0

quality curves, corresponding to specified air quality standards, intersect. Since we expect the optimal $x(t)$ to lie on or in the vicinity of the vertex point, we can safely replace the air quality constraints $g \leqq g^*$ by the following expressions:

$$x_1(t) = k_1(t) \tag{10}$$

$$x_1(t) + k_2(t)x_2(t) = k_3(t) \tag{11}$$

where $k_i(t)$ are constants for year t so that (10) and (11) appropriately represent the iso-air quality curves (as illustrated in Figure 1).

SOURCE INVENTORY AND RELATED DATA

In order to solve the optimization problem we need to estimate source inventory data for 1973-1975 as well as available control methods, their costs, and emission reduction characteristics. The most complete published contaminant emission inventories for Los Angeles are those of Roth et al. (1973) [see also Roberts et al. (1971, 1973)] and Trijonis (1972). The former study presents an emissions inventory for 1969, with particular emphasis on the detailed spatial and temporal distribution of motor vehicle emissions. The inventory compiled by Trijonis is concerned mainly with the total county-wide emissions and the number of potentially controllable pieces of emitting equipment (automobiles, power plant boilers, etc.) in Los Angeles County in 1975. Both inventories rely in part on data published by the Los Angeles County Air Pollution Control District (1969, 1971). As mentioned, the reactive hydrocarbon emission estimates of Trijonis are now known to be too low since the Environmental Protection Agency has determined that only C_1 to C_3 paraffins, acetylene and benzene can be considered truly unreactive. With this modification, and adjustment for the different areas covered (Trijonis' inventory is for Los Angeles County whereas that of Roth et al. is for a region encompassing part of Orange County in addition to Los Angeles County), the two inventories have been reconciled for 1973-1975.

An inventory of sources based on estimates of growth rates given by Trijonis is presented in Table 1. Uncontrolled emission levels from these sources are given in Table 2. Control methods, costs and emission reduction characteristics are presented in Table 3.

Table 4 shows the estimated supply limits of natural gas for Los Angeles in 1973-1975. Finally, Table 5 presents the projected uncontrolled emissions from all sources in 1973-1975 in Los Angeles. The data in Tables 1 to 4 provide all the necessary inputs to the optimal control evaluation.

The system (1) to (5) with (10) and (11) constitutes a linear programming problem with unknowns $w(t)$, $x(t)$, $t = 1,2,3$. For the present Los Angeles problem, $w(t)$ is 31-dimensional and $x(t)$ is 2-dimensional. The dimensions of $y^0(t)$, $s(t)$, $l(t)$ and $g^*(t)$ are 2, 23, 1, and 2, respectively.

The results obtainable from the problem being considered are:

1. The sources to be controlled, the amount of control on each source and the emissions from each source when controlled;
2. The cost incurred by each of the sources and the total control cost;
3. The time table for the allocation of control methods over the various years; and
4. The effect of the air quality path on the total cost of control.

Table 4. Estimated Availability of Natural Gas in Los Angeles County

Uses	1973	1974	1975
1. Total amount of natural gas available, barrels/yr.	1.16×10^8	1.13×10^8	1.10×10^8
2. Residential and non-power plant use, barrels/yr.	8.23×10^7	8.55×10^7	8.9×10^7
3. Amount available to power plants and fleet vehicles	3.37×10^7	2.75×10^7	2.1×10^7

Table 5. Estimated Precontrolled Emissions of Reactive Hydrocarbons and Oxides of Nitrogen in Los Angeles County from All Sources, tons/day

Pollutants	1973	1974	1975
RHC	1095	1041	975
NO_x	820	807	794

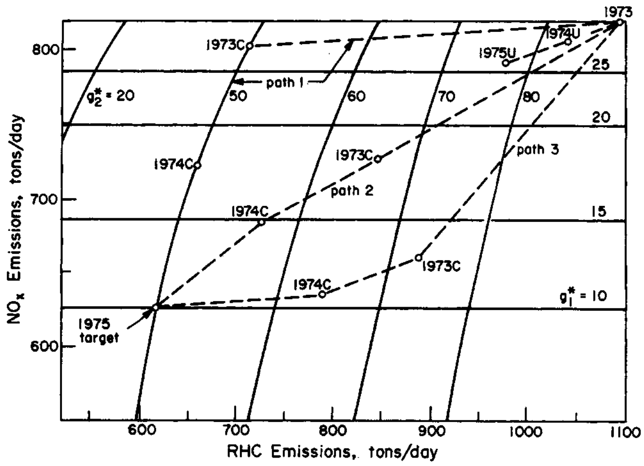

Fig. 2. Air quality paths depicting uncontrolled (U) and controlled (C) emissions of RHC and NO_x in Los Angeles County from 1973 to 1975.

EVALUATION OF THREE AIR QUALITY PATHS FROM 1973-1975

The problem we have formulated yields the least cost set of control measures over a given number of years subject to meeting prescribed air quality constraints in each of the years. The constraints chosen correspond to the three air quality paths shown in Figure 2. Path 1 was selected to exhibit extreme control of RHC emissions during the first year, followed by mostly NO_x control in 1974 and 1975. Path 3 was chosen to reflect the other extreme of extensive NO_x control in 1973, followed by mostly RHC control in 1974 and 1975. Path 2 is based on rather uniform reductions in both pollutants each year over the three-year period. We stress that we have chosen these paths somewhat arbitrarily, the main purpose being to examine the effect of the air quality path on the distribution of control measures and the total cost of control. Thus, we are more interested in the sensitivity of the total cost of control to various air quality objectives than in the precise specification of control levels on the sources.

The result of the linear programming solution for path 2 only is summarized in Table 6. Table 7 compares the yearly costs along paths 1, 2, and 3. Finally, Table 8

TABLE 6. DISTRIBUTION OF CONTROL ALONG AIR QUALITY PATH 2 IN FIGURE 2

				New controls instituted in							
1973				1974				1975			
Control methods used	Control units	NO_x reduct., tons/day	RHC reduct., tons/day	Control methods used	Control units	NO_x reduct., tons/day	RHC reduct., tons/day	Control methods used	Control units	NO_x reduct., tons/day	RHC reduct., tons/day
15	361	0	25.6	8	7	21.8	0	12	37	11.5	0
16	10,800	0	40.3	15	8	0	0.55	15	9	0	0.6
17	1	0	132	16	200	0	0.75	16	300	0	1.1
18	24	0	21.6	18	25	0	22.5	18	26	0	23
19	25	0	17.8	20	1.1×10^6	17.9	65.2	23	4.14×10^5	23.6	2.2
23	6.1×10^5	47.6	4.5	31	650	0	0.94	27	3.45×10^5	51.1	35.9
26	4×10^5	45.1	0								
31	6,350	0	9.2								
New reductions, tons/day		92.7	251.0			39.7	89.7			86.2	62.8
Reductions carried over, tons/day						80.4	228.8			81.4	296.2
Total reductions, tons/day		92.7	251.0			120.1	318.5			167.6	359.0

summarizes the major sources to be controlled and the predominant methods to be used. (The entries in Table 8 are listed in decreasing order of importance.)

DISCUSSION OF THE RESULTS

The validity of the results obtained in a least-cost exercise such as just described depends strongly on the accuracy of the emission inventory and the emission level/air quality model used. The various parameters required in the formulation of the problem are:

1. Source parameters: source units in each year (including growth rate projections); spatial and temporal distributions of emissions; uncontrolled emission levels.
2. Control method parameters: control cost per unit of control; pollutant emission reduction per unit of control.
3. Atmospheric parameters: frequencies of meteorological conditions, reaction rate constants, radiation intensity.

In general, of the parameter groups listed above, control method parameters are known most accurately. Source parameters, while inherently more difficult to determine than those relating to control, can be determined accurately if an emission inventory is carefully compiled for the region of interest. The greatest uncertainty in source parameters probably lies in the estimation of growth rates in future years. The group of inputs to the entire control strategy determination that are known least accurately are usually those relating to meteorology. The set of meteorological inputs needed depends, of course, on the emission level/air quality model used.

In an empirical model, such as that employed above, based on correlations of past air quality monitoring data with emission levels, meteorological parameters need not be explicitly specified, rather they appear implicitly in the air quality/emission level correlation. The major difficulty in using a statistical model of the type employed here is choosing the air quality measure. Possible choices of such a measure include, for example, the maximum hourly-average daily concentration at a certain location in the airshed and the maximum eight-hour-average daily concentration over all monitoring stations in the airshed. The choice of a measure of air quality is somewhat arbitrary, and the correlation with area-wide emission levels will vary depending on this choice. The statistical model used here correlates maximum hourly-average concentrations of NO_2 and O_3 in downtown Los Angeles with the total daily mass emissions of NO and hydrocarbons in Los Angeles County. Thus, this model is not able to be used to evaluate control strategies for meeting air quality objectives at locations other than downtown Los Angeles. In addition, in the application of this model, all sources of a certain type are controlled uniformly regardless of their location in the airshed. (To the extent that the main source is motor vehicles, this assumption is reasonable.) The statistical model's advantage, as compared to a physical model, is that a criterion based on the number of days per year that a certain standard is violated can be easily used.

TABLE 7. COSTS OF CONTROL ALONG THE THREE AIR QUALITY PATHS IN FIGURE 2, MILLIONS OF DOLLARS

Air quality path	1973	1974	1975	Total
1	69.9	70.0	71.3	211.2
2	12.4	22.6	35.4	70.4
3	20.2	26.2	31.4	77.8

TABLE 8. MAJOR SOURCES TO BE CONTROLLED IN LOS ANGELES COUNTY

Major pollutant reduction	Major sources to be controlled	Major control methods
NO_x	1. 1966-1969 vehicles	Vacuum spark advance disconnect and tuning
	2. 1971-1974 fleet vehicles	To burn natural gas
	3. Power plants	To burn natural gas
RHC	1. pre-1966 vehicles	Capacitor discharge ignition optimization systems installed
	2. Organic solvent users.	Further restriction on organic solvent users
	3. Service stations	Vapor recycle systems installed
	4. Degreasers	Substitution of 1,1,1, Trichloroethane for Trichloroethylene

Finally we must consider the sensitivity of the control results (for example, the total control cost, the level of employment of the various methods) to the various parameters in the problem. We have already discussed the accuracy of the classes of inputs to the problem formulation. A related, but separate, question involves the amount of influence exerted by each parameter on the outcome of the problem. From the results of studies with the control problem we are able to draw the following qualitative conclusions.

The solution of a given control problem appears to be most sensitive to the chosen air quality constraints. When the air quality constraint approaches values representing ambient air quality standards, the cost of control begins to increase sharply. The parameters describing the cost per unit control method appear in the cost function J. Those parameters representing the source magnitudes appear on the right-hand sides of (2) to (4), while those describing control methods and their effectiveness enter the left-hand sides of (2) to (4). It is well known in the theory of linear programming (Gale, 1960) that the minimal cost J is a continuous function of both the costs per unit control and the right-hand sides of the linear programming constraints (2) to (4) but not necessarily of the parameters appearing on the left hand sides of these constraints. Therefore, a slight change in the value of one source parameter (say, the number of 1970 model automobiles) will produce only a slight change in the minimal cost J. However, a slight change in one of the control parameters (say, the reduction of NO_x emissions from one pre-1966 automobile by the institution of one vacuum spark advance disconnect system) may produce a significant change in the minimal cost. Therefore, from the point of view of sensitivity, the control method parameters are the most important (at a specified level of air quality). Fortunately, the control parameters are probably also among the most accurate of those data needed.

For the overall problem the relation of meteorological parameters to others in the system in terms of sensitivity will, of course, depend on the emission level/air quality model used in evaluating the control measures. However, since reported airshed modeling studies (Reynolds et al., 1973a,b,c) have shown that the predictions of physical models are quite sensitive to meteorological inputs, we would expect that meteorological parameters would play an important role in the final results of the overall problem.

In conclusion, it appears that sensitivity to parameter variations should not be a critical problem for the control method/emission level calculation, since the crucial parameters can be rather accurately estimated. On the other hand, improving the ability to mathematically model atmospheric pollutant behavior appears to be a rewarding goal for further study.

SUMMARY

We have considered the problem of determining a least-cost set of air pollutant source control measures that achieve specified levels of air quality. The problem is formulated in general terms, and the formulation is applied to the determination of control measures for reactive hydrocarbon and oxides of nitrogen emissions in Los Angeles County over the period 1973-1975. It is shown that the number of days per year that the NO_2 and O_3 primary standards are violated can be reduced from the estimated 1975 level of 26 and 77 days, respectively, to 10 and 50 days at a cost of approximately 70 million dollars over the period 1973-1975.

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